Part 1

0606/22/F/M/19

1. **Solutions to this question by accurate drawing will not be accepted.**

The points $A(3, 2)$, $B(7, -4)$, $C(2, -3)$ and $D(k, 3)$ are such that CD is perpendicular to AB. Find the equation of the perpendicular bisector of CD.

[6]

$m_{AB} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-4-2}{7-3}$

$= -\frac{6}{4} = -\frac{3}{2}$

$m_{AB} \times m_{CD} = -1$

$m_{CD} = \frac{2}{3}$

$\frac{3+3}{k-2} = \frac{2}{3}$

$6 \times 3 = 2k - 4$

$2k = 18 + 4$

$2k = 22$

$k = 11$

midpt of $CD = \left(\frac{2+11}{2}, \frac{-3+3}{2}\right)$

$= \left(\frac{13}{2}, 0\right)$

$y = -\frac{3}{2}x + c$

$0 = -\frac{39}{4} + c$

$c = \frac{39}{4}$

$y = -\frac{3}{2}x + \frac{39}{4}$

$4y = -6x + 39$

0606/21/M/J/19

2. **Solutions to this question by accurate drawing will not be accepted.**

   The points *A* and *B* have coordinates (*p*, 3) and (1, 4) respectively and the line *L* has equation 3*x* + *y* = 2.

   (i) Given that the gradient of *AB* is $\frac{1}{3}$, find the value of *p*. [2]

$$m_{AB} = \frac{4-3}{1-p}$$

$$\frac{1}{1-p} = \frac{1}{3}$$

$$1-p = 3$$

$$1-3 = p$$

$$p = -2$$

   (ii) Show that *L* is the perpendicular bisector of *AB*. [3]

$$y = -3x + 2$$

$$m_L = -3$$

$$m_{AB} \times m_L = \frac{1}{3} \times -3 = -1$$

$\therefore$ AB $\perp$ Line L

$$\text{midpt } AB = \left(\frac{-2+1}{2}, \frac{3+4}{2}\right)$$

$$= \left(-\frac{1}{2}, \frac{7}{2}\right)$$

$$y = -3 \times -\frac{1}{2} + 2$$

$$= \frac{3}{2} + 2 = \frac{7}{2}$$

$\therefore$ L is the perpendicular bisector of AB.

   (iii) Given that $C(q, -10)$ lies on *L*, find the value of *q*. [1]

$$y = -3x + 2$$

$$-10 = -3q + 2$$

$$-12 = -3q$$

$$q = 4$$

   (iv) Find the area of triangle *ABC*. [2]

$$\text{Area of } \triangle ABC = \frac{1}{2} \left| \begin{array}{cccc} A & B & C & A \\ -2 & 1 & 4 & -2 \\ 3 & 4 & -10 & 3 \end{array} \right|$$

$$= \frac{1}{2} \left| (-8 - 10 + 12) - (3 + 16 + 20) \right|$$

$$= \frac{1}{2} \left| -6 - 39 \right|$$

$$= 22.5 \text{ unit}^2$$

The Maths Society

0606/23/M/J/19

3. The points A, B and C have coordinates (4, 7), (-3, 9) and (6, 4) respectively.

   (i) Find the equation of the line, L, that is parallel to the line AB and passes through C. Give your answer in the form $ax + by = c$, where a, b and c are integers.

   [3]

$$m_{AB} = \frac{9-7}{-3-4} = \frac{2}{-7} = m_L$$

$$y = -\frac{2}{7}x + c$$

$$4 = -\frac{12}{7} + c$$

$$c = 4 + \frac{12}{7} = \frac{40}{7}$$

$$y = -\frac{2}{7}x + \frac{40}{7}$$

$$7y = -2x + 40$$

$$2x + 7y = 40$$

   (ii) The line L meets the x-axis at the point D and the y-axis at the point E. Find the length of DE.

$y = 0$ $\quad$ $x = 0$

   [2]

$$7y = -2x + 40$$

$$0 = -2x + 40$$

$$2x = 40$$

$$x = 20$$

$D(20, 0)$

$$7y = 0 + 40$$

$$y = \frac{40}{7}$$

$E(0, 40/7)$

$$DE = \sqrt{\left(\frac{40}{7}\right)^2 + (20)^2}$$

$$= 20.8$$

The Maths Society

4. **Do not use a calculator in this question.**

The curve $xy = 11x + 5$ cuts the line $y = x + 10$ at the points *A* and *B*. The midpoint of *AB* is the point *C*. Show that the point *C* lies on the line $x + y = 11$.

[7]

$x(x+10) = 11x + 5$

$x^2 + 10x = 11x + 5$

$x^2 - x - 5 = 0$

$$x = \frac{1 \pm \sqrt{1+20}}{2}$$

$$x = \frac{1 \pm \sqrt{21}}{2}$$

$y = x + 10$

$= \frac{1+\sqrt{21}}{2} + \frac{10^{\times 2}}{1_{\times 2}}$

$= \frac{21+\sqrt{21}}{2}$

$y = x + 10$

$= \frac{1-\sqrt{21}}{2} + \frac{10^{\times 2}}{1_{\times 2}}$

$= \frac{21-\sqrt{21}}{2}$

A $\left(\frac{1+\sqrt{21}}{2}, \frac{21+\sqrt{21}}{2}\right)$

B $\left(\frac{1-\sqrt{21}}{2}, \frac{21-\sqrt{21}}{2}\right)$

midpt (c): $\left(\frac{1}{2}, \frac{21}{2}\right)$

$x = 1/2$

$x + y = 11$

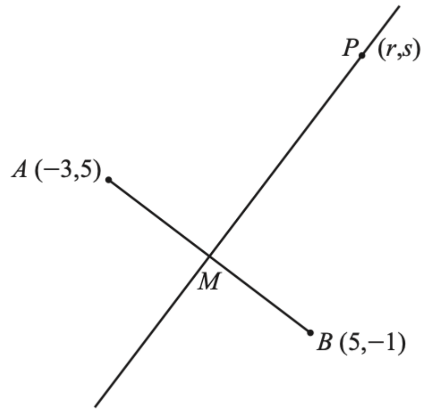
$\frac{1}{2} + y = 11$

$y = 11 - \frac{1}{2} = \frac{21}{2}$ $\therefore$ C lies on the line.

The Maths Society

0606/21/O/N/19

5.



The diagram shows the points A (–3, 5) and B (5, –1). The midpoint of AB is M and the line PM is perpendicular to AB. The point P has coordinates (r, s).

a. Find the equation of the line PM in the form $y = mx + c$, where m and c are exact constants.

[5]

$$\text{midpt } AB = \left(\frac{-3+5}{2}, \frac{5-1}{2}\right)$$

$$M = (1, 2)$$

$$m_{AB} = \frac{-1-5}{5+3} = \frac{-6}{8} = -\frac{3}{4}$$

$$m_{pm} = \frac{4}{3}$$

$$y = \frac{4}{3}x + c$$

$$2 = \frac{4}{3} + c$$

$$c = \frac{6}{3} - \frac{4}{3} = \frac{2}{3}$$

$$y = \frac{4}{3}x + \frac{2}{3}$$

b. Hence find an expression for s in terms of r.

[1]

$$\frac{4}{3} = \frac{s-2}{r-1}$$

$$4r - 4 = 3s - 6$$

$$4r - 4 + 6 = 3s$$

$$4r + 2 = 3s$$

$$s = \frac{4r+2}{3}$$

The Maths Society

c. Given that the length of *PM* is 10 units, find the value of *r* and of *s*.

[5]

$$PM = \sqrt{(s-2)^2 + (r-1)^2}$$

$$10^2 = \left(\frac{4r+2}{3} - 2\right)^2 + r^2 - 2r + 1$$

$$100 = \left(\frac{4r+2-6}{3}\right)^2 + r^2 - 2r + 1$$

$$100 = \left(\frac{4r-4}{3}\right)^2 + r^2 - 2r + 1$$

$$100 = \frac{16r^2 - 32r + 16}{9} + r^2 - 2r + 1$$

$$900 = 16r^2 - 32r + 16 + 9r^2 - 18r + 9$$

$$0 = 25r^2 - 50r - 875$$

$$0 = r^2 - 2r - 35$$

$r = 7$ or $r = -5$ (reject)

$$s = \frac{4r+2}{3} = 10$$

$P(7, 10)$